What is the Surface Area of a Sears-Haack Body?

This question comes up from time to time. It seems quite straightforward. To compute the surface area of a body of revolution, you must integrate the function

$$S = 2\pi \int r\sqrt{1 + (dr/dx)^2}dx$$

from nose to tail.
For a Sears-Haack body (fixed length and volume)

$$r = [4x(1-x)]^{\frac{3}{4}}$$

$$\frac{dr}{dx} = 3\left[4x(1-x)\right]^{-\frac{1}{4}}(1-2x)$$

where r is radius / max.radius and x is the distance from nose / body length.

Now, the only problem with this is that no one seems to be able to carry out this integration symbolically. This includes the powerful symbolic math aids. However, it is rather straightforward to perform the integration numerically and I have a numerics page at http://www.pdas.com/areash2.xml that describes the mathematical and programming techniques. For those of who who just want the answer, get the factor from the following table for the fineness ratio (length/max diameter) of your body. Then compute the surface area from

$$S = (2\pi r_{max} L) factor$$

f	factor
2	0.7811
4	0.7358
6	0.7265
8	0.7232
10	0.7217
12	0.7208
14	0.7203
16	0.7200
18	0.7197
20	0.7196
22	0.7195
24	0.7194
26	0.7193
28	0.7192
30	0.7192
40	0.7191
50	0.7190

The quantity in parentheses is the surface area of a cylinder of the same length and diameter as the Sears-Haack body. So, if you just remember that the area is about 72 percent of this quantity, that will be good enough for most purposes.

areash.tex

Last modified 22 January 2003 by Ralph Carmichael (ralph@pdas.com)

Public Domain Aeronautical Software
P.O. Box 1438
Santa Cruz CA 95061