

ALGORITHM FOR CALCULATING COORDINATES OF CAMBERED NACA AIRFOILS AT SPECIFIED CHORD LOCATIONS

by Ralph L. Carmichael¹
 Public Domain Aeronautical Software
 P.O. Box 1438
 Santa Cruz CA 95061 USA

ABSTRACT

The equations for the NACA 4-digit and 4-digit-modified sections are in algebraic form and easily incorporated into various geometrical procedures that define a vehicle and any necessary flow field grid. The 6-series and 6A-series airfoils are more complex because they are developed by conformal mapping procedures. Even though there are computer programs available (refs 1-3) that can produce a large table of points on the surface of the airfoil, there is a frequently expressed desire for an algorithm that will calculate the upper and lower ordinates and slopes of a cambered airfoil at a specified chord location that requires no interpolation on the part of the user. The purpose of this paper is to present such an algorithm and describe subroutines that may be used for these calculations. A public domain computer program incorporating these procedures has been written and may be downloaded from the author's web site. This program is modular, allowing its internal procedures to be used in other programs.

INTRODUCTION

In spite of the advanced airfoils available to the modern designer, the NACA series of airfoils continue to be of interest. Computer programs were written in 1974-1975 (Ref 1-2) and updated in 1996 (Ref. 3) that enable a user to produce a table of surface coordinates of an airfoil with thickness and camber from the NACA families.

These original programs have been included in the collections of public domain aeronautical software distributed by the author (Ref 4). Users of these programs identified two areas where the programs did not totally satisfy their needs. Firstly, many users wanted to specify the chord position of the airfoil and compute the upper and lower ordinates of the surface directly above and below this chord point. The output from the programs of refs 1-3 displaces the upper and lower x-coordinates because the thickness is applied perpendicular to the mean line. Secondly, many users wanted subroutines that performed these calculations that could be incorporated in a larger analysis or design program. The monolithic structure of the programs of Refs 1-3 discouraged any such use. In order to satisfy these user requirements, the present algorithm was developed.

THICKNESS OF 6- AND 6A-SERIES AIRFOILS

The NACA 6-series airfoils are calculated by a nonlinear mapping of a unit circle by a four-step algorithm that uses a pair of functions defined on $[0, \pi]$ named ψ and ϵ that were chosen to satisfy a prescribed velocity distribution about the airfoil. The definition of the ψ and ϵ functions is described in refs 7-8. Each of the five members of the 6-series family and the three members of the 6A-series family has its own ψ and ϵ functions. These functions are multiplied by a scale factor to produce airfoils of various thickness to chord ratios. The mapping is shown in figure 1. A given value

¹ Senior Member, AIAA.

of the scale factor is used to multiply both basic parameters giving new values of the ψ and ϵ functions that will be used in the mapping. A given value of scale factor will produce a certain thickness to chord ratio of the airfoil in the normalized physical plane. It is not known in advance just what thickness will result from a given value of the scale factor. The algorithms of references 1 and 3 use an iterative procedure to determine the scaling factor required to achieve an airfoil of a given thickness.

The algorithm used in the present method is based upon a study of the scaling factor required to achieve a given thickness. Calculations were made of the thickness resulting from a given value of scale factor for each of the eight airfoil families. The results are shown as Figure 2. The dependency is somewhat nonlinear but easily fitted as a polynomial with four coefficients. The fitting is done on the data as if scale factor c is a function of t/x .

$$c = K_1(t/c) + K_2(t/c)**2 + K_3(t/c)**3 + K_4(t/c)**4$$

	K_1	K_2	K_3	K_4
63	8.1827700	1.3776209	-0.0928517	7.5942563
64	4.6535511	1.0380630	-1.5041794	4.7882784
65	6.5718716	0.4937629	0.7319794	1.9491474
66	6.7581414	0.1925377	0.8128826	0.852090
67	6.6272890	0.0989966	0.9675977	0.9053758
63 A	8.1845925	1.0492569	1.3115094	4.4515579
64 A	8.2125018	0.7685596	1.4922345	3.6130133
65 A	8.2514822	0.4656936	1.5013018	2.0908904

Now, for a specified family and thickness, the thickness distribution may be determined without iteration. From the thickness, the scale factor is computed from the polynomial function shown above. Then, the scale factor is used to multiply the basic values of the ψ and ϵ functions for this airfoil family. These scaled ψ and ϵ functions are used in mapping the z -plane to the z' -plane shown in Figure 1. The Joukowski function

$$\zeta = z' + 1/z'$$

then maps the z' -plane into the zeta-plane and these results are normalized so that the leading edge is at $x=0$

and the trailing edge is at $x=1$.

ORDINATES AT A SPECIFIED X-COORDINATE

The calculation as described in the previous paragraph is usually done by selecting a uniformly distributed set of points on the circle in the z -plane. The resulting points in the normalized zeta-plane will be nicely distributed, but not, in general, at the points desired by the user. The solution is to interpolate in the tables of computed points to get the desired x -coordinates. Many people have encountered difficulties in doing this near the leading edge because the slope of the thickness distribution becomes infinite there. The preferred technique is to define a parametric variable and express both x and y as functions of this variable. Arc length is a rather obvious choice and arc length can be approximated by inscribed arc length quite well if there are sufficient points in the data set.

In figure 3, the x - and y -coordinates of the scaled airfoil are shown as a function of arc length. The arc length, s , is taken as zero at the trailing edge, proceeds forward along the upper surface reaching a value slightly greater than one at the leading edge and then proceeds rearward along the lower edge to its final value of slightly greater than two at the trailing edge. The x - and y -coordinates are seen to be smooth functions of s with smooth derivatives whose magnitude never exceeds one (see Figure 4 for derivatives). With no troublesome infinities, the interpolation is quite straightforward. For a given x , the value of s is computed by the inverse of the x vs. s function and from this value of s , the corresponding value of y is computed. Cubic splines are used for the interpolation.

The algorithms described in the paragraphs above enable us to produce a portable computational procedure. Given an airfoil family, max thickness and a table of x -coordinates along the chord, a table of corresponding y -coordinates is produced. In fact, the first and second derivatives may also be computed. This satisfies the second of the user needs outlined in the introduction. This algorithm has been coded as Subroutine Thickness6 in the Fortran module.

CAMBERED AIRFOILS AT SPECIFIED X-COORDINATES

Having the thickness and y -coordinate of the mean line at each desired x -coordinate enables one to compute the table of ordinates of the upper and lower surfaces of the

cambered airfoil. Users are frequently distressed to find that the output tables are not at these x-coordinates, but at slightly different values. The reason for this can be seen from Figure 5. As explained in Refs 1-6, the thickness is always applied perpendicular to the mean line and the computed points on the upper and lower surfaces are shifted from the original values on the chord.

$$\begin{aligned}x_{upper} &= x_{mean} - t \sin \theta \\y_{upper} &= y_{mean} + t \cos \theta \\x_{lower} &= x_{mean} + t \sin \theta \\y_{lower} &= y_{mean} - t \cos \theta\end{aligned}$$

In these equations, t is one-half the thickness. The result is shown in figure 6. For a given distribution of points along the chord, the black points are those that would be produced by the programs of ref 1-4. To satisfy the user's needs, the white points should be output by the computing procedure.

Again, the solution is interpolation using both x and y as functions of a parametric variable, namely arc length, s. For a given x, one finds the value of s that corresponds to this value of x on the upper surface and obtains y at the same s. This process is repeated for the lower surface. The derivative dy/dx is computed by computing dx/ds and dy/ds at the interpolation point and computing $dy/dx = (dy/ds) / (dx/ds)$. The algorithm described here is quite general and does not require the thickness profile or the mean line to belong to the NACA families. Figures 7 and 8 illustrate the interpolation process for a cambered airfoil.

COMPUTER CODE

A number of functions and subroutines have been written and checked that implements the algorithms described above. These procedures and the mean line subroutines that are essentially unchanged have been coded in modern Fortran. The six-series and six-A-series thickness calculations use the COMPLEX data type of Fortran and thereby are much simpler to understand compared to explicitly working out the real and imaginary parts of the functions. The procedures have been encapsulated in a module called NacaAuxiliary. Users who wish to incorporate these algorithms into their own computer programs can simply include this module. Some of the public procedures in

this module are:

```
MeanLine2(xmax,ymax,x,ym,ymp)
MeanLine6(a,x,ym,ymp)
ComboMeanLine6(a,cl,x,ym,ymp)
MeanLine6Modified(cl,x,ym,ymp)
Thickness4(tc,x,y,yp,ypp)
Thickness4SharpTE(tc,x,y,y,yp,ypp)
Thickness4modified(tc,xc,x,y,yp,ypp)
Thickness6(family,tc,x,y,yp,ypp)
CombineThicknessAndCamber(x,yt,ym,ymp,
    xupper,yupper,xlower,ylower)
```

In the subroutines above, x is a given array of points in fraction of chord. The arrays y, yp, ypp are output arrays of the same dimension as x with corresponding values of y-coordinates, derivatives and second derivatives. The arrays ym and ymp are output arrays of the same dimension as x with corresponding values of mean line y-coordinates and slopes. a is the chordwise extent of uniform load and cl is design lift coefficient. Consult the user guide accompanying the program for fuller explanations of these and other parameters associated with the subroutines and functions.

A program that uses this module has been written to emulate the original programs called NACA4 and NACA6 that were based on the 1974-1975 programs of references 1,2. This program allows a user to select an airfoil of a given family and thickness and combine it with an arbitrary mean line and print out a table of upper and lower ordinates and slopes at a user-specified set of chord locations.

All of the code mentioned above has been declared open-source or public domain and is available from the author's web site at <http://pdas.com>.

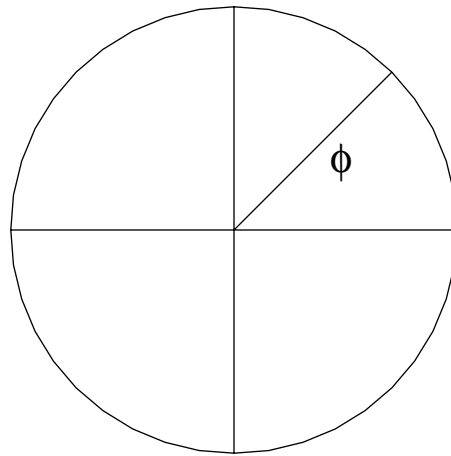
CONCLUSIONS

An algorithm that generates a table of coordinates of a cambered NACA airfoil at a user-specified set of chord locations has been described. The procedure is modular and may be easily be incorporated into other analysis or design programs. The algorithm has been coded and compared with existing programs. The public domain routine is coded in Fortran and may be downloaded from the author's web site.

REFERENCES

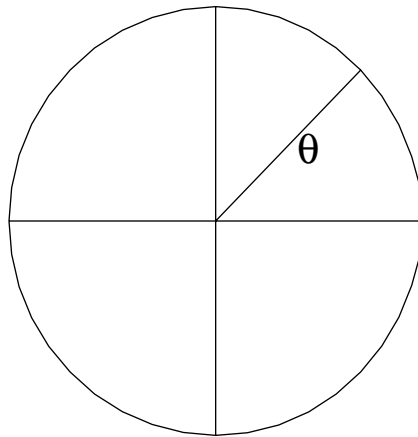
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AIRFOIL DEVELOPMENT BY TRANSFORMATION



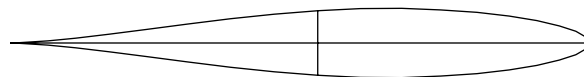
z-plane

$$z' = z * \exp \{(\psi - \psi_0) + i\epsilon\}$$



z'-plane

$$\zeta = z' + 1/z'$$



zeta-plane



normalized zeta-plane

Figure 1 - Airfoil Development by Transformation

VARIATION OF T/C WITH SCALE FACTOR

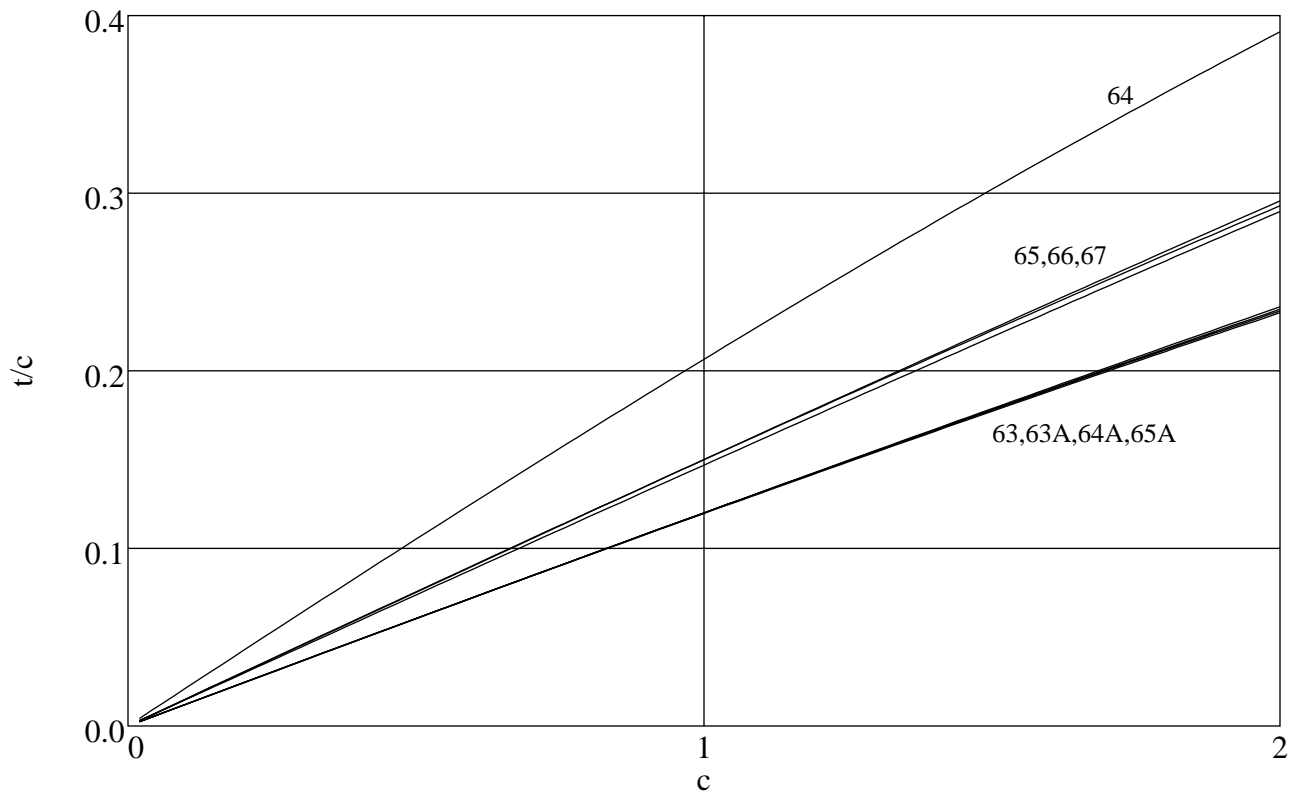


Figure 2 - Variation of t/c with scale factor

63A018 AIRFOIL IN PARAMETRIC COORDINATES

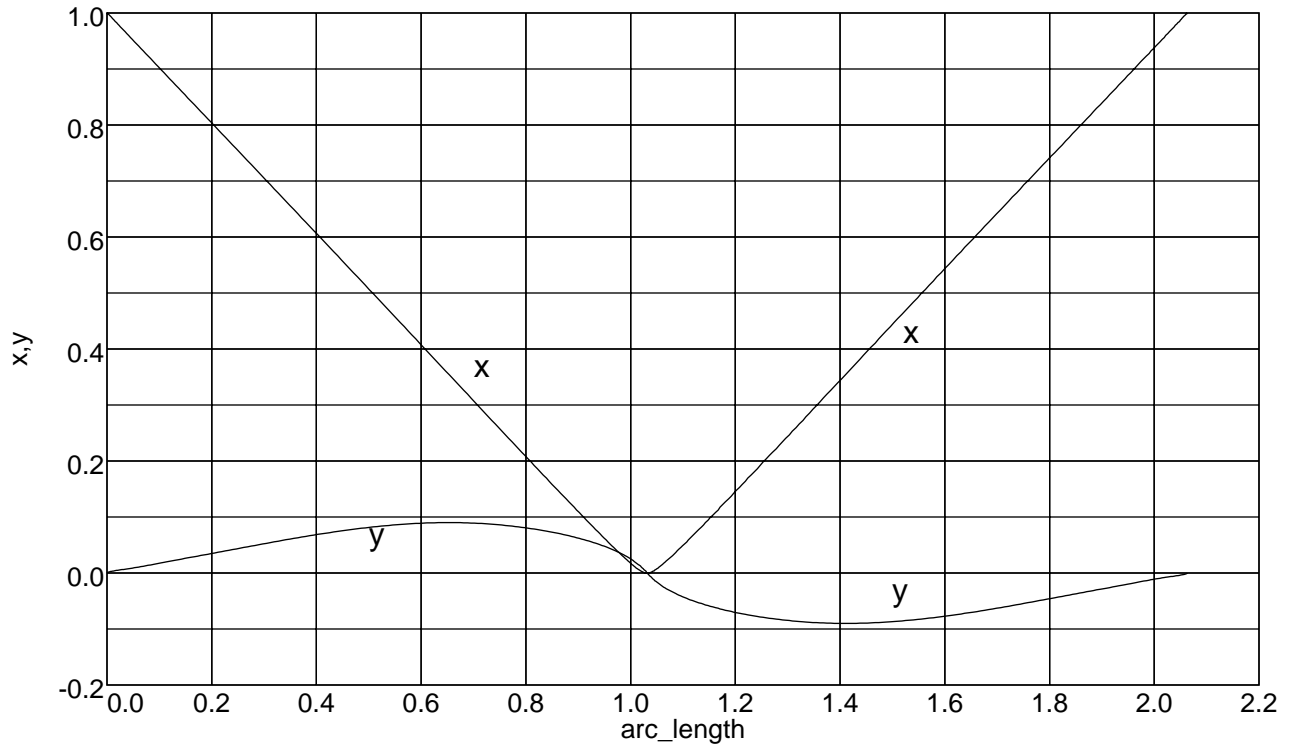


Figure 3 - Airfoil in Parametric Coordinates

DERIVATIVES IN PARAMETRIC COORDINATES

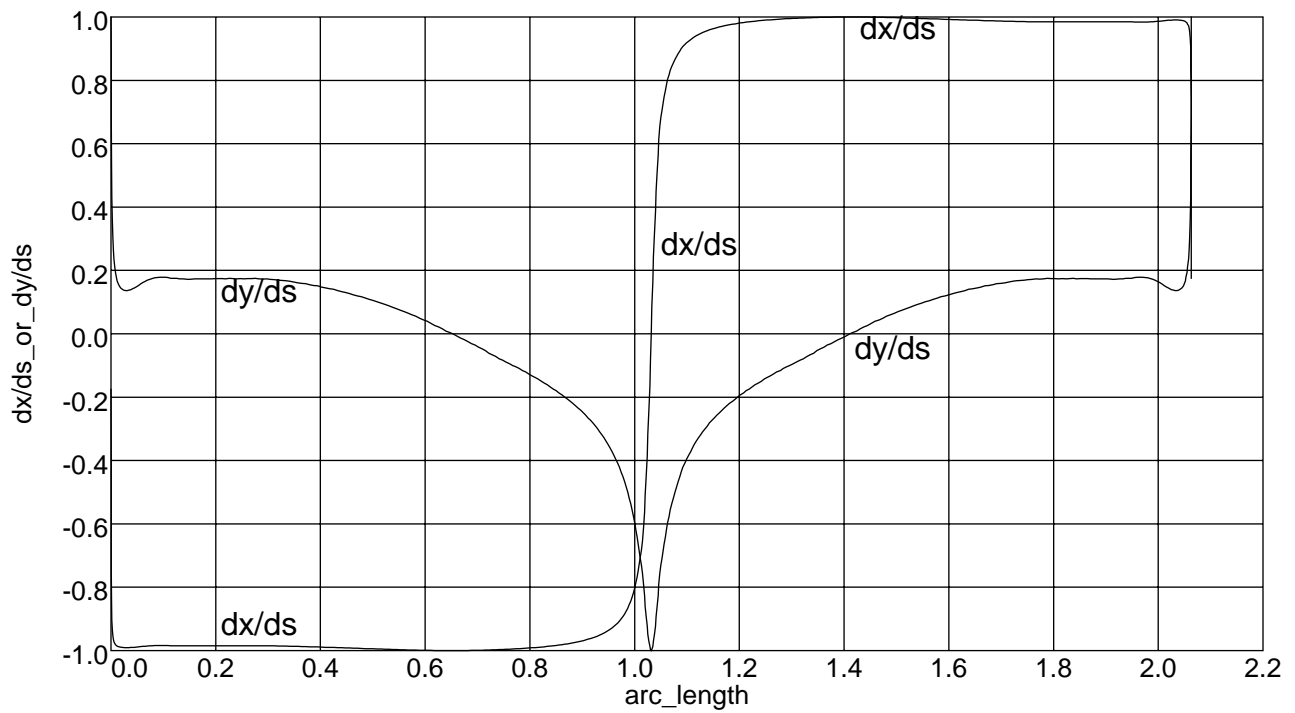


Figure 4 - Derivatives of Airfoil in Parametric Coordinates

COMBINING MEAN LINE AND THICKNESS

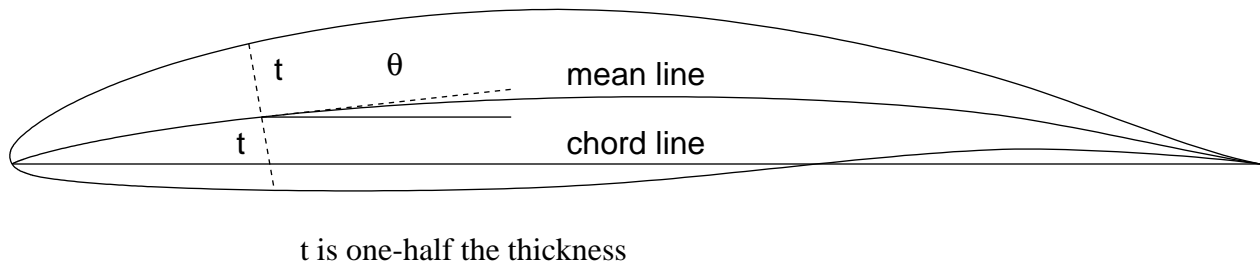


Figure 5 - Combining Mean Line and Thickness

OUTPUT POINTS

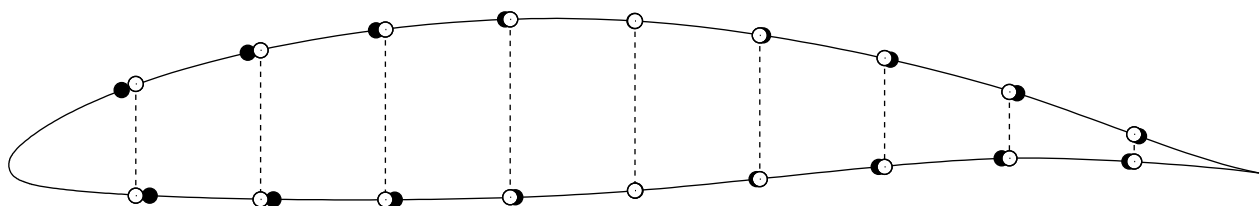


Figure 6 - Output Points

65-415 AIRFOIL IN PARAMETRIC COORDINATES

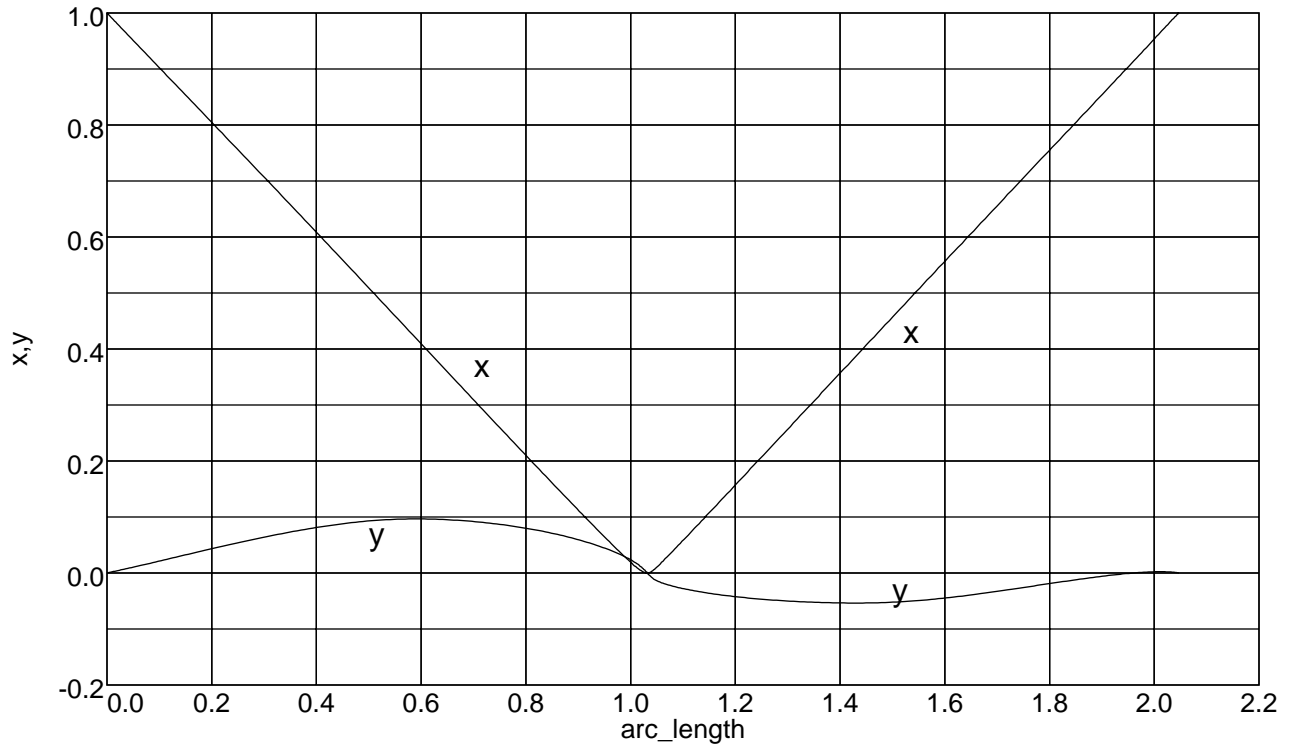


Figure 7 - 65-415 Airfoil in Parametric Coordinates

DERIVATIVES IN PARAMETRIC COORDINATES

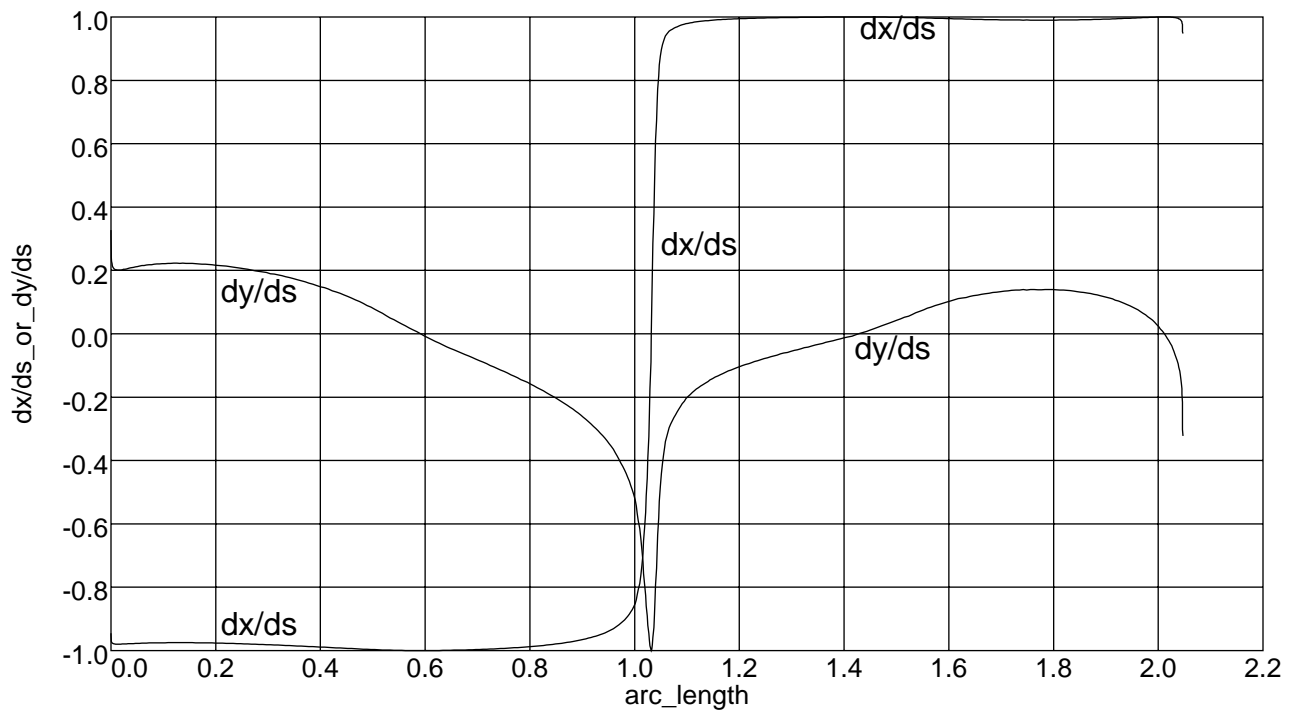


Figure 8 - Derivatives of Cambered Airfoil in Parametric Coordinates