2. METHOD

OUTLINE OF METHOD

The zero-lift drag of arrow wings at supersonic speeds is computed by superposition of source and sink distributions in the plane $z=0$ of Figure 1. A single source solution of the linearized differential equation for the velocity potential satisfies a boundary condition for an elementary wedge in a supersonic stream. Hence an arrow-shaped wing of wedge section results from a distribution over the planform area of sources whose constant strength is proportional to the wedge streamwise slope. The linear basic differential equation then permits the superposition of source or sink distributions which satisfy the wing surface slope condition over successive segments. In general, a final distribution of sources is required along and downstream of the trailing edge to satisfy the condition that the flow returns to the free-stream direction. However, the restriction to a straight trailing edge which intersects the wing centerline at a right angle at most (that is, a delta planform) obviates this last distribution. The surface pressure associated with each source distribution is integrated over all non-zero slopes of the wing to obtain the total drag.

BASIC FORMULAE

The drag coefficient for wings of the type shown in Figure 1 can be obtained by extension of the equations given by Puckett[1] to the case $d \neq 0$. The initial wing section slope is

$$\lambda_1 = \frac{\tau}{2(1-r)} \qquad (1)$$

and the final wing section slope is

$$\lambda_3 = -\frac{\tau}{2(a-d)} \qquad (2)$$

All the wing geometric parameters are defined in Figure 1. The total drag coefficient in general is composed of the following terms

$$C_D = C_{D_{IA}} + C_{D_{IB}} + C_{D_{IC}} + C_{D_{ID}} + C_{D_{IIIA}} + C_{D_{IIIB}} + C_{D_{IIIC}} + C_{D_{IIID}} \qquad (3)$$

where, for example, $C_{D_{IA}}$ denotes the drag coefficient contributed over region I by the source distribution starting at point A and covering the wing area. Clearly there is no drag contribution from region II. Now defining

$$n = k/\beta = k/(M^2-1)^{\frac{1}{2}} \qquad (4)$$

the terms in Equation 3 are expressed as follows:

$$C_{D_{IA}} = \frac{2\tau^2}{\beta\pi}\frac{1}{(1-r)^2}G(n,r) \qquad (5)$$

LIST OF SYMBOLS

a = dimensionless distance (see Figure 1)
b = dimensionless distance (see Figure 1)
c = over-all length (see Figure 1)
C_D = wave drag coefficient based on wing area
d = dimensionless distance (see Figure 1)
F = basic drag function defined by Equation 13
G = basic drag function defined by Equations 14-16
k = tan Λ
M = free-stream Mach number
n = k/β (relative sweepback)
r = dimensionless distance (see Figure 1)
β = $\sqrt{M^2-1}$
Λ = leading edge sweep angle (see Figure 1)
λ = wing section surface slope streamwise (see Figure 1)
τ = dimensionless thickness (see Figure 1)

$$C_{D_{IB}} = -\frac{2\tau^2}{\beta\pi}\frac{r}{(1-r)^2}F(n,r) \tag{6}$$

$$C_{D_{IC}} = -\frac{2\tau^2}{\beta\pi}\frac{a}{(1-r)(a-d)}\left[F(n,a)-F\left(rn,\frac{a}{r}\right)\right] \tag{7}$$

$$C_{D_{ID}} = \frac{2\tau^2}{\beta\pi}\frac{d}{(1-r)(a-d)}\left[F(n,d)-F\left(rn,\frac{d}{r}\right)\right] \tag{8}$$

$$C_{D_{IIIA}} = -\frac{2\tau^2}{\beta\pi}\frac{1}{(1-r)(a-d)}[G(n,d)-G(n,a)] \tag{9}$$

$$C_{D_{IIIB}} = \frac{2\tau^2}{\beta\pi}\frac{r}{(1-r)(a-d)}\left[G\left(rn,\frac{d}{r}\right)-G\left(rn,\frac{a}{r}\right)\right] \tag{10}$$

$$C_{D_{IIIC}} = \frac{2\tau^2}{\beta\pi}\frac{a}{(a-d)^2}G\left(an,\frac{d}{a}\right) \tag{11}$$

$$C_{D_{IIID}} = -\frac{2\tau^2}{\beta\pi}\frac{d}{(a-d)^2}F\left(an,\frac{d}{a}\right) \tag{12}$$

The two basic drag functions, F and G, given in References 1 and 3, in terms of general variables m and s, are

$$F(m,s) = \frac{1-s}{1+s}\left[\frac{\log ms}{\sqrt{(ms)^2-1}} + \frac{1}{\sqrt{m^2-1}}\log\frac{sm^2-1+\sqrt{(m^2-1)(m^2s^2-1)}}{m(1-s)}\right] \tag{13}$$

if $ms > 1$

$F = 0$ when $ms \leq 1$ $\quad F(m,1) = 0$

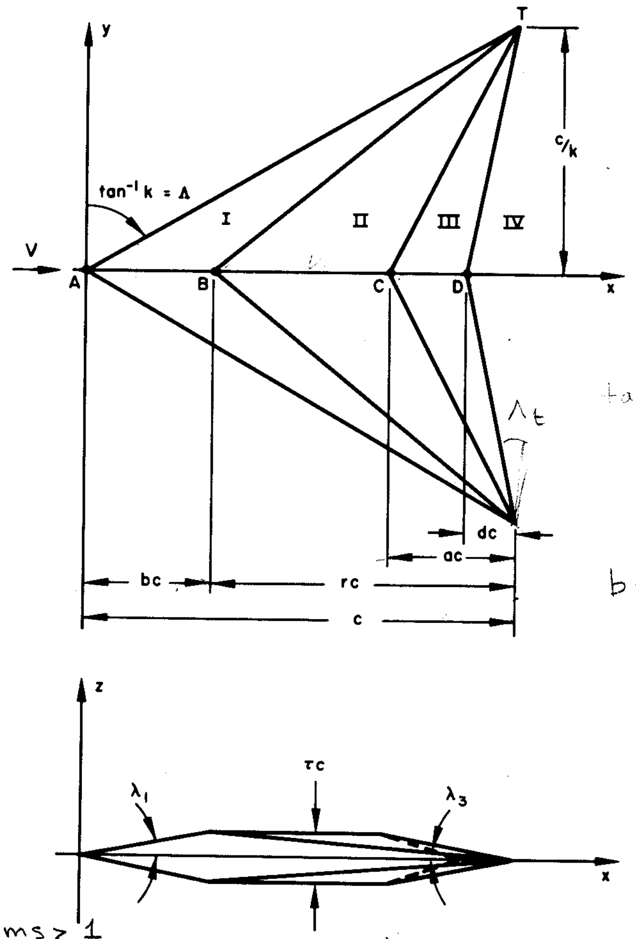

FIGURE 1. GEOMETRY OF ARROW WING

$\tan \Lambda_t =$

$b = 1 - r$

For $ms > 1$ and $m > 1$, $G(m,s)$ is given by

$$G(m,s) = \frac{1-s}{1+s}\left[\frac{\log m + s\cosh^{-1} m}{\sqrt{m^2-1}} + \frac{1}{\sqrt{(ms)^2-1}}\log\left(1+\frac{2\sqrt{(ms)^2-1}}{m(1-s)+\sqrt{m^2-1}-\sqrt{(ms)^2-1}}\right)\right] \tag{14}$$

For $ms < 1$ and $m > 1$

$$G(m,s) = \frac{1-s}{1+s}\left[\frac{\log m + s\cosh^{-1} m}{\sqrt{m^2-1}} + \frac{2}{\sqrt{1-(ms)^2}}\tan^{-1}\left(\frac{\sqrt{1-(ms)^2}}{m(1-s)+\sqrt{m^2-1}}\right)\right] \tag{15}$$

For $ms < 1$ and $m < 1$

$$G(m,s) = \frac{1-s}{1+s}\left[\frac{s\cos^{-1} m}{\sqrt{1-m^2}} + \frac{1}{\sqrt{1-(ms)^2}}\left(\frac{\pi}{2}+\sin^{-1} ms\right)\right] \tag{16}$$

Limiting forms:

$$G(1,s) = \frac{1-s}{1+s}\left[s + \frac{1}{\sqrt{1-s^2}}\left(\frac{\pi}{2}+\sin^{-1} s\right)\right]$$

$G(1,1) = 0$

$G(m,1) = 0$

$G(0,s) =$