Computing the Wave Angle of an Attached Shock Wave

The goal is a procedure that computes the wave angle for a given deflection angle and Mach number. The first consideration is that a solution may not exist for many combinations of input variables. This suggests that this procedure should be coded as a subroutine rather than as a function and that an error code be returned to signify when no solution is to be found.

The subroutine should have the following calling sequence
SUBROUTINE ObliqueShock(mach,delta,theta,errCode)
where mach and delta are the input values of Mach number and deflection angle. Since this procedure is an internal subroutine rather that a piece of interface, delta should probably be given in radians. Theta is the computed value of wave angle, again in radians, and errCode is an output code that indicates the action taken by the subroutine. A value of 0 is usually taken as sucess and that the value of theta returned by the subroutine is to be believed. Other values of errCode could be 1 for subsonic Mach number and 2 for delta greter than the maximum deflection angle for this Mach number and 3 for a negative deflection angle.

The routine MaxRampAngle developed on the page
http://www.pdas.com/maxramp2.xml
can be used to determine whether an attached shock is permissible. For completeness, we should check that Mach is supersonic and that the deflection angle is positive and less than $\pi/2$.

The deflection angle, delta, for a given wave angle, theta, and a given Mach number M is governed by

$$\cot\delta = \tan\theta\left[\frac{(\gamma+1)M^2}{2(M^2\sin^2\theta-1)} - 1\right]$$

or

$$\tan\delta = \frac{2\cot\theta(M^2\sin^2\theta-1)}{2+M^2(\gamma+1-2\sin^2\theta)}$$

The computational technique to be employed here is Newton's method. We are looking for a root of the function defined as deflection angle minus desired deflection angle as a function of wave angle. In order to use Newton's method, we need to be able to compute the function and its derivative at any given wave angle.

I will not say that it is obvious or even trivial to get the derivative of this equation with respect to theta, but with a bit of determination and perhaps access to a symbolic mathematics program, you should be able to come up with

$$(1+\cot^2\delta)\frac{d\delta}{d\theta} = \frac{2M^2\sin^2\theta}{(M^2\sin^2\theta-1)^2} + \frac{1}{\cos^2\theta} - \frac{1}{2}(\gamma+1)M^2\frac{1+\tan^2\theta}{M^2\sin^2\theta-1}$$

When computing with Newton's method, it is important to use a good starting value. The wave angle must lie between the extreme values of the Mach angle and $\pi/2$. But, if the starting value is near $\pi/2$, the solution will converge to a strong shock solution. To insure that we converge to the weak shock, it is best to start with the Mach angle plus a tiny amount to insure that the floating point calculation has not left us just a bit less than the Mach angle.

Then apply Newton's formula

$$x_{new} = x_{old} - f(x_{old})/f'(x_{old})$$

iteratively until convergence is achieved.

When using an iterative procedure such as Newton's method, you must be able to decide when to halt the iteration. This is usually done by testing to see whether the new value of the root is sufficiently close to the old one. Alternately, one can test whether the function value is sufficiently small and accept the current value as the root. However, for this problem, we will avoid all of these numerical details and always carry out four iterations. I have done extensive testing and have found that four iterations will converge to double precision (64-bit) accuracy for any input variable.

The source code for the subroutine and a small test program can be downloaded from http://www.pdas.com/programs/oshock.f90

The results you should get from running this test program can be downloaded from http://www.pdas.com/programs/oshock.out

oshock2.tex

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